



WEST BENGAL STATE UNIVERSITY
B.Sc. Honours/Programme 1st Semester Examination, 2022-23

MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.*

Answer Question Number 1 and any five from the rest

1. Answer any *five* questions from the following: 2×5 = 10

(a) Examine whether the limit $\lim_{x \rightarrow 3} \frac{[x]}{x}$ exists, where $[x]$ represents the greatest integer less or equal to x .

(b) $f(x) = \begin{cases} x+1 & \text{when } x \leq 1 \\ 3-ax & \text{when } x > 1 \end{cases}$

For what value of a , will f be continuous at $x=1$.

(c) For the function $f(x) = |x|$; $x \in \mathbb{R}$ show that $f'(0)$ does not exist.

(d) Show that the function $f(x) = 4x^2 - 6x - 11$ is increasing at $x = 4$.

(e) Find the point on the curve $y = x^3 - 6x + 7$ where the tangent is parallel to the straight line $y = 6x + 1$.

(f) Find the asymptotes of the curve $xy^2 - yx^2 - (x + y + 1) = 0$.

(g) Examine the continuity of the function at $(0, 0)$

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

(h) Show that the function $f(x, y) = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$ is homogeneous in x and y . Find its degree.

(i) If $u = x \log y$, then show that $u_{xy} = u_{yx}$.

2. (a) If f is an even function and $f'(0)$ exists, then show that $f'(0) = 0$. 4

(b) Discuss the continuity of f at $x=1$ and $x=2$ where $f(x) = |x-1| + |x-2|$. 4

3. (a) If $x + y = e^{x-y}$, show that $\frac{d^2y}{dx^2} = \frac{4(x+y)}{(x+y+1)^3}$ 4

(b) State and prove Lagrange's Mean Value Theorem. 4

4. (a) Find the slope of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ at the point (x_1, y_1) and hence obtain the equation of the tangent at that point. 4
- (b) Verify Rolle's theorem for the function $f(x) = x\sqrt{4-x^2}$ is $0 \leq x \leq 2$. 4
5. (a) Expand $f(x) = \sin x$ as a series of infinite terms. 5
- (b) If $y = \frac{x}{x+1}$, show that $y_5(0) = 51$. 3
6. (a) If $f(x) = \log \frac{\sqrt{a+bx} - \sqrt{a-bx}}{\sqrt{a+bx} + \sqrt{a-bx}}$, find for what values of x , $\frac{1}{f'(x)} = 0$. 4
- (b) Prove that $\lim_{h \rightarrow 0} \frac{f(a+h) - 2f(a) + f(a-h)}{h^2} = f''(a)$, provided that $f''(x)$ is continuous. 4
7. (a) Find the maxima and minima, if any, of $\frac{x^4}{(x-1)(x-3)^3}$. 2+2
- (b) Determine the values of a, b, c so that $\frac{a \sin x - bx + cx^2 + x^3}{2x^2 \log(1+x) - 2x^3 + x^4}$ may tend to a finite limit as $x \rightarrow 0$, and determine this limit. 3+1
8. (a) If $lx + my = 1$ is a normal to the parabola $y^2 = 4ax$, then show that $al^3 + 2alm^2 = m^2$. 4
- (b) If the tangent at (x_1, y_1) to the curve $x^3 + y^3 = a^3$ meets the curve again in (x_2, y_2) , show that $\frac{x_2}{x_1} + \frac{y_2}{y_1} = -1$. 4
9. (a) Prove that the asymptotes of the curve $x^2 y^2 = a^2(x^2 + y^2)$ form a square of side $2a$. 4
- (b) Show that for an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the radius of curvature at an extremity of the major axis is equal to the half of the latus rectum. 4
- 10.(a) If V is a function r alone, where $r^2 = x^2 + y^2 + z^2$, show that 4
- $$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{d^2 V}{dr^2} + \frac{2}{r} \frac{dV}{dr}$$
- (b) If $y = f(x+ct) + \phi(x-ct)$, show that $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. 4